Final Review Review

The final is cumulative, so you should be prepared for anything we have done all quarter. In addition to the topics mentioned here, you should be study the topics from the midterm review sheets.

1. Area between curves

Given 2 curves, you should be able to calculate the area between them. This might involve finding points of intersection or discovering an enclosed region

Examples:

Find the area of the region bounded by $x = 1 - y^2$ and $x = y^2 - 1$

Find the area of the region bounded by $y = 8 - x^2$, $y = x^2$, $x = -3$, and $x = 3$

Find the area of the region bounded by $y = \sin(\pi x/2)$, and $y = x$

Find the area of the region bounded by $y = 1 - \cos x$ and $y = \cos x$ between 0 and π

Find the area of the region bounded by $y = |x|$ and $y = x^2 - 2$

Find the area of the region bounded by $x = 2y^2$ and $x = 4 + y^2$

Find the area of the region bounded by $y = \frac{1}{x}$, $y = \frac{1}{x^2}$ and $x = 2$

2. Volume

Given a solid, you should be able to find the cross-sectional area to get a typical slice and then take an integral to get the volume of the shape.

Examples:

Calculate the volume of a pyramid of height 12 ft and a square base with sides 4ft.

Calculate the volume of the solid defined as follows. Take the region bounded by the x-axis and the parabola $y = 4 - x^2$. The solid is above this region, vertical cross-sections of the solid which intersect the (x,y)-plane parallel to the x-axis are semi-circles.

Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x-1}$, $y = 0$ between $x = 2$ and $x = 5$ about the x-axis

Find the volume of the solid obtained by rotating the region bounded by $y = x^{2/3}$, $y = 0$ between $x = 0$ and $x = 1$ about the y-axis

Find the volume of the solid obtained by rotating the region bounded by $y = 2x^2 - x^3$ and $y = 0$ about the y-axis

Find the volume of the solid obtained by rotating the region bounded by $y = \cos(\pi x/2)$, $y = 0$, on $0 \leq x \leq 1$ about the y-axis.

Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about the y-axis

Find the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 0$, $x = 1$ and $x = 4$ about the y-axis

Find the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 0$, $x = 1$ and $x = 4$ about the line $x = 1$

Find the volume of the solid obtained by rotating the region bounded by $y = 4x - x^2$, $y = 8x - 2x^2$ about the line $x = -2$

3. Work

Work is the integral of force $W = \int_a^b F(x)dx$.

Example: An 8ft chain weighs 16 lbs and hangs off the side of a building. Find the work required to lift the chain up to the top of the building.

Example: A cable with mass 2kg/m is used to lift a 100kg mass up 300m. How much work is done in total.

Problems involving pumping water out of a tank are good but require a picture. Problems 21-24 in the book are good. Also, if you would like more practice, consider if the tank starts half full (according to height not according to volume) or $\frac{1}{4}$ full or whatever.

4. Average Value of a function

The average value of a function f is simply $\frac{1}{b-a}\int_a^b f(x)dx$

Example:

Find the average value of $f(x) = \frac{1}{(x+1)(x-2)}$ between 1 and 4

5. Arc Length

We have several formulas for calculating arc length, make sure you know them.

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Examples:

1) Calculate the arc length of $f(x) = \frac{1}{12}x^3 + x^{-1}$ over[1,3]

1) Calculate the arc length of $f(x) = \ln(\cos x)$ over$[0, \frac{\pi}{3}]$

6. Surface Area

The surface area of a function rotated about an axis is $\int_a^b 2\pi r * (arclength)$

Example:

Find the surface area of the cone obtained by rotating the line $y = 2x$ about the x-axis for $0 \leq x \leq 4$.

Example:

Find the surface area of the surface obtained by rotating $y = \sqrt{1+4x}$ for $1 \leq x \leq 5$ about the x-axis

Example

Find the surface area of the surface obtained by rotating $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ for $1 \leq x \leq 2$ about the y-axis

Example

Find the surface area of the surface obtained by rotating $y = \sqrt[3]{x}$ for $1 \leq y \leq 2$ about the y-axis