Midterm 2 Review

The second midterm will focus on techniques of integration. This includes u-substitution (see midterm 1 review), integration by parts, trig substitution, partial fractions, and improper integrals

1. integration by parts

For integration by parts, you are generally looking for a part for which the derivative is simpler, and the rest has an integral which is not more complicated.

$$\int x \cos x dx$$

we may have to do parts multiple times,

$$\int x^2 e^x dx$$

$$\int_1^2 x^4 (\ln x)^2 dx$$

we may have to do a u-sub in addition to parts,

$$\int_{\pi/4}^{\pi/3} \sec^2 x \ln(\tan x) dx$$

$$\int x^5 e^{x^2} dx$$

There are also cases in which we have a part which gets much simpler when the derivative is taken, even though the rest gets slightly more complicated when an integral is taken.

$$\int \tan^{-1} x dx$$

$$\int \ln x dx$$

$$\int \sin^{-1} x dx$$

and we may have problems that you do parts a few times, it cycles back to itself and then we have that funny trick

$$\int e^{2x} \sin 7x dx$$

2. trig substitution

In these problems you will have to make a substitution that puts trig into the problem.

$$\int \frac{x^3}{\sqrt{16-x^2}}dx$$

$$\int \frac{1}{t^3\sqrt{t^2-1}}dx$$

$$\int \frac{\sqrt{1+x^2}}{x}dx$$

$$\int_0^{2/3} x^3\sqrt{4-9x^2}dx$$

$$\int x\sqrt{1-x^4}dx$$

$$\int \frac{1}{(5-4x-x^2)^{5/2}}dx$$

3. partial fractions

If you have to integrate a quotient of 2 polynomials, they you will most likely want to use the method of partial fractions. Don't forget to make sure the degree of the denominator is higher than the numerator, otherwise you have to use long division. Also, remember that there are special rules if there are repeated factors or quadratic factors.

$$\int \frac{dx}{x^2-7x+10}$$

$$\int \frac{x^3+1}{x^2-4}dx$$

$$\int \frac{3x-9}{(x+2)(x^2+x-2)}dx$$

$$\int \frac{18}{(x+3)(x^2+9)}dx$$

$$\int \frac{1}{(x+5)(x^2+4x-5)}dx$$

Don't forget, you may have to use u-sub first, then partial fractions,

$$\int \frac{1}{x\sqrt{x+1}}dx$$

or there can be a problem that looks like partial fractions but is simply u-sub.

$$\int \frac{2x+3}{x^2+3x+4}dx$$

4. improper integrals

Remember that anytime we are dealing with improper integrals, it is possible that the relevant limit will not exist, and therefore the integral diverges.

$$\int_{-\infty}^{-1} \frac{dx}{x}$$

$$\int_{2}^{\infty} \frac{dx}{x^2}$$

Sometimes you have to use L'Hôpital's Rule

$$\int_{0}^{\infty} xe^{-x}dx$$

Be careful of the tricky problems with discontinuities in the middle

$$\int_{0}^{3} \frac{dx}{x-2}$$