Midterm 1 Review

This review sheet is to remind you of what we have done and what are potentially topics for the exam. In addition to the problems that I provide here, a great source of practice material is the homework, the quizzes, and the text.

Derivatives

At this point, you should be experts at derivatives. Obviously there won't be a question on the exam to just find a derivative, but they will show up within other problems. Below is a standard list of derivatives that you should know very well.

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln a$$

$$\frac{d}{dx}lnx = \frac{1}{x}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(f(x) * g(x)) = f(x)\frac{d}{dx}g(x) + \frac{d}{dx}f(x) * g(x)$$

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) * g'(x)$$

Trig

You should have the following trig Identities memorized.

$$\tan x = \frac{\sin x}{\cos x}$$
$$\sec x = \frac{1}{\cos x}$$
$$\csc x = \frac{1}{\sin x}$$
$$\cot x = \frac{\cos x}{\sin x}$$
$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\sin 2x = 2 \sin x \cos x$$

Also, you should know the basic values of the trig functions. Know the values of all of them for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and the equivalent values in the other 3 quadrants.

The following identities I would like you to be able to use, however, I don't expect them to be memorized, they will be provided to you on the exam.

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$
$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$
$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$
$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$
$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)$$

Summation Notation

You need to know how to use summation notation, in particular you need to know how to use the following three formulas. The first should be memorized, but the other 2 will be provided on the exam.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

These formulas show themselves while we solve integrals using Riemann sums.

Definition of Integral

A good question would ask you to solve an integral using the definition of integrals. This would require setting up a Reimann sum, using summation formulas, and taking a limit. Depending on how the question is asked, you may or may not have to actually solve the integral.

Examples:

1) Evaluate $\int_0^3 x + 1dx$ using the definition of derivative

2) Set up but do not evaluate a Reimann Sum for $\int_2^7 \sin^2(2x) + x^3dx$.

(Notice that this one looks worse, but it's actually easier since you don't have to evaluate anything.)

Approximation and Error Bounds

You should know how to approximate the area under the curve of a function using midpoint rule, and you should know how to calculate the error bound.

Integrals as Area

Also, remember that integrals can be thought of as (signed) area under a curve.

Example:

Evaluate $\int_{-1}^1 \sqrt{1-x^2}dx$. Notice this is a half circle, and you know that area.

In this particular example, if you do not notice that it is a half circle, you can still solve it by the method of trig substitution, however, that would take considerably more time.

Properties of Integral

You should know the basic properties of the integral stated in section 5.2, and where applicable, be able to draw a quick sketch justifying the property

(1) $\int_a^b cdx = c(b-a)$

(2) $\int_a^b [f(x) + g(x)] = \int_a^b f(x)dx + \int_a^b g(x)dx$

(3) $\int_a^b cf(x)dx = c\int_a^b f(x)dx$
(4) $\int_a^c f(x) + \int_c^b f(x)dx = \int_a^b f(x)dx$
(5) $\int_a^b f(x)dx = -\int_b^a f(x)dx$
(6) If $f(x) \geq 0$ for $0 \leq x \leq b$, then $\int_a^b f(x)dx \geq 0$
(7) If $f(x) \geq g(x)$ for $0 \leq x \leq b$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$
(8) if $m \leq f(x) \leq M$ for $a \leq x \leq b$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

Fundamental Theorem

There are 2 parts to the fundamental theorem. In class, we went over the proof of the 2 parts. In addition to understanding the statement of the theorems, you should understand the basic ideas of the proofs.

You should be familiar with the function $g(x)$ described in this section and you should know how it relates to $f(x)$.

Examples:

1. If $f(x) = \int_0^x \sin^2 t dt$ then find $f'(x)$.
2. If $f(x) = \int_{x^2}^{\sin x} (t^3 + \ln t)dt$ find $f'(x)$.
3. Given a function for f or g find the other one, or perhaps find specific values of the other one.

Indefinite vs. Definite Integrals

Know the difference between indefinite integrals and definite ones. Know when and when not to use +C

U-substitution

There will certainly be a u-sub question on the exam. There are a lot of good u-substitution problems in the text, I will put a few here.

$$\int \frac{(\ln x)^2}{x} dx$$

$$\int_1^2 \frac{e^{1/x}}{x^2} dx$$

$$\int \frac{1+x}{1+x^2} dx$$

$$\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$$

U-substitution with a lot of trig

These are integrals that are several trig functions multiplied together. There are basically 4 types:

you have sin's and cos's and both are to an even power. In this case you want to use half angle formula

$$\int \sin^2 x dx$$

$$\int \sin^2 x \cos^2 x dx$$

you have sin's and cos's and at least one is to an odd power.

$$\int \sin^4 x \cos^5 x dx$$

you have sec's and tan's (or csc's and cot's, they are functionally the same)

$$\int_0^{\pi/3} \tan^5 x \sec^4 x dx$$

$$\int \cot^3 x \csc x dx$$

you also might have to manipulate things to get them into the correct form

$$\int \cos^2 x \sin x \sin(2x) dx$$

The last type of question involves not having the same argument within the trig function.

$$\int \sin(5x)\sin(2x) dx$$

You might have to combine techniques

$$\int \csc^2 x \cos x \sin^3(\csc x)\cos^2(\csc x) dx$$

Even and Odd Functions

You should know what it means for a function to be even or odd, and you should know how to exploit that fact when calculating it's integral

1. Formulas

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$|E_M| \leq \frac{K\,(b-a)^3}{24n^2}$$