

## MATH 122B: MIDTERM

- (1) Show that if  $f$  is holomorphic in  $D$ , then

$$f'(z_0) = \frac{df}{dz}(z_0) = \frac{\partial f}{\partial x}(z_0) = -i \frac{\partial f}{\partial y}(z_0)$$

- (2) Compute the integral  $\int_{-\infty}^{\infty} \frac{\cos(2x)dx}{x^4 + 1}$ . Make sure to show that the answer you obtain is a real number.
- (3) Find  $\int_{|z|=2} \frac{dz}{(z^{2016} + 1)(z - 3)(z - 4)}$  in closed form (not as a summation).
- (4) Show that if  $f$  is holomorphic on  $\mathbb{C}$  and  $|f(z)| \leq |z|$  for all  $z \in \mathbb{C}$ , then  $f = Az$  for some constant  $A$ .
- (5) Compute the Laurent series of  $e^{z+\frac{1}{z}}$  centered at  $z = 0$ . Then compute the integral  $\int_{|z|=1} e^{z+\frac{1}{z}}$ . Show that the magnitude of your answer is a finite number.