# FINAL STUDY GUIDE

## 1. Concepts

**1.1. First order linear.** Solution by characteristic ODE/ change of variable

**1.2. Heat Equation.**

- Heat kernel solution for infinite and half-life
- With initial conditions
- Homogeneous and nonhomogeneous
- Separable solutions for bounded domain. This will have boundary conditions as well as initial conditions
- When is maximum principle applicable?

**1.3. Wave Equation.**

- d'Alembert solution for the infinite line, even/odd extension solution for half-line
- homogeneous/nonhomogeneous.
- two initial conditions
- Separable solution for bounded interval. This will have boundary conditions.

**1.4. Laplace Equation.**

- Maximum principle.
- Separable Solutions.

## 2. Some Exercises

(1) Solve the eigenvalue problem

$$y'' + \lambda y = 0 \quad \text{on } [0, L]$$

with
  (a) Dirichlet boundary $y(0) = y(L) = 0$
  (b) Neumann boundary $y'(0) = y'(L) = 0$.

(2) Show that the eigenvalue is nonnegative for Neumann boundary and strictly positive for Dirichlet boundary. Note that $y = 0$ is the trivial case, which we disregard.

(3) Write the Laplacian $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ in polar coordinates.

(4) Solve Laplace's equation

$$\Delta u = 0$$

for the unit ball $x^2 + y^2 = 1$ with Dirichlet boundary condition. (Hint: maximum principle).

(5) Assuming the maximum principle for harmonic functions, show that the solution to the Dirichlet problem of the Poisson equation

$$\begin{cases} \Delta u = f & \text{in } D \\ u = h & \text{on } \partial D. \end{cases}$$

where $f$ and $h$ are appropriate given functions.

(6) Section 4.1, 1–6

(7) Section 4.2, 1–4