FINAL

(1) Find the general solution, then attempt to find a particular solution. Determine if there is a unique solution, no solution, or infinitely many solutions of

$$yu_x + x^2 u_y = xy,$$

(a) $u(x,y) = 4x$ on the curve $y = \frac{1}{3}x^{\frac{3}{2}}$.
(b) $u(x,y) = x^2$ on the curve $3y^2 = 2x^3$.
(c) $u(x,y) = \sin(x)$ on the line $y = 0$.

(2) Compute all the eigenvalues of the Neumann problem

$$\begin{cases} y'' + \lambda y = 0 & \text{on } 0 < x < L \\ y'(0) = y'(L) = 0. \end{cases}$$

Then use this to solve the heat equation using separation of variables

$$\begin{cases} u_t - u_{xx} = 0 & \text{on } 0 < x < L \\ u_x(0,t) = u_x(L,t) = 0 \\ u(x,0) = \phi(x). \end{cases}$$

(3) Similar to 2, compute the eigenvalues and solve

$$\begin{cases} u_{tt} - c^2 u_{xx} & \text{on } 0 < x < L \\ u_x(0,t) = u(L,t) = 0. \end{cases}$$

Note that the boundary conditions are mixed now.

(4) Let $D \subset \mathbb{R}^2$ be a bounded domain and

$$\Delta u(x,y) \geq 0, \quad \text{on } D$$

Prove that the maximum value of u is attained on the boundary of D. Hint: Consider some modification of the type $u + \varepsilon(x^2 + y^2)$.