MIDTERM 1

(1) Solve the first order PDE

$$\begin{cases} -2u_x + u_y - yu = 0 \\ u(x,0) = x^2. \end{cases}$$

(2) Attempt to find a particular solution of

$$3yu_x - 2xu_y = 0.$$

Determine if there is a unique solution, no solution, or infinitely many solutions for auxiliary conditions

(a) $u(x,y) = x^2$ on the line $y = x$.
(b) $u(x,y) = 1 - x^2$ on the line $y = -x$.
(c) $u(x,y) = 2x$ on the ellipse $2x^2 + 3y^2 = 4$.

(3) Show that the general solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$ is given by

$$u(x,t) = f(x - ct) + g(x + ct)$$

for sufficiently differentiable functions f and g by considering the change of variables

$$s = x - ct$$
$$r = x + ct.$$

(4) Solve the initial value problem

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{on } -\infty < x < \infty, t > 0 \\ u(x,0) = 0 \\ u_t(x,0) = \psi(x). \end{cases}$$

where

$$\psi(x) = \begin{cases} 1 & \text{on } |x| \leq 1 \\ 0 & \text{on } |x| > 1. \end{cases}$$

You may leave your solution in terms of ψ. Using this solution, answer the following

(a) Will the wave ever return to its original state at $x = 0$?
(b) What happens at each point x when $t \to \infty$?
(c) For some positive time $t > 0$, is there a location x such that $u(x,t) = 0$?