MIDTERM STUDY GUIDE

Study the homework problems too.

1. CHAPTER 1

1.1. **First Order Linear.**
Know how to find general solutions via method of characteristic. For example:

- $-2u_x + u_y - yu = 0$.
- $xu_x + yu_y + 2 = 0$
- $\sigma^2 u_\sigma + \sigma\gamma u_\gamma + \sigma u = \sigma - \gamma$.

Also know how to analyze the solutions. For the Cauchy problem, determine if there is a unique solution, no solution, or infinitely many solutions:

$$u_x - 6u_y = y$$

with data

- $u(x, y) = e^x$ on the line $y = -6x + 2$.
- $u(x, y) = 1$ on the line $y = -x^2$.
- $u(x, y) = -4x$ on the line $y = -6x$.

2. CHAPTER 2

2.1. **Wave Equation.**
Know how to find solutions of the Cauchy problem to wave equations via d'Alembert's formula and **be able to analyze the solution i.e. is it increasing? decreasing? oscillating?** For example, solve the Cauchy problem

$$\begin{cases} u_{tt} - 9u_{xx} = 0 & \text{on } -\infty < x < \infty, t > 0, \\ u(x,0) = \cos(x) \\ u_t(x,0) = \sin(2x) \end{cases}$$

Also be comfortable with changing variables. Solve the wave equation $u_{tt} - c^2 u_{xx} = 0$ by considering a change of variables $s = x - ct$ and $r = x + ct$.

Also look at exercise 8,9,10,11 in Section 2.1.

2.2. **Diffusion Equation.**
Hopefully we will get to maximum principle, look at exercises in 2.3