

MATH 124B: MIDTERM

(1) Let $X \in C^2$ on (a, b) . Assuming symmetric boundary conditions, i.e. $X'(x)X(x)\Big|_a^b = 0$, prove that the eigenvalue of $X'' + \lambda X = 0$ on (a, b) is non-negative.

(2) Solve the problem

$$\begin{cases} u_{tt} = c^2 u_{xx} \\ u(0, t) = u(L, t) = 0 \\ u(x, 0) = x, \\ u_t(x, 0) = 0. \end{cases}$$

(3) Find the sum $\sum_{n \text{ odd}} \frac{1}{n^2}$ using any method.

(4) Let

$$f_n(x) = \frac{n}{1 + n^2 x^2} - \frac{n-1}{1 + (n-1)^2 x^2}.$$

Show that $\sum_{n=1}^{\infty} f_n(x)$ converges point-wise to 0. Show that it does not converge uniformly and in L^2 .

(5) Solve the equation $u_{xx} + u_{yy} = 1$ in the annulus $a < r < b$ with $u = 0$ on the boundary $r = a$ and $r = b$.