# MATH 147A: HOMEWORK 1

**Due Thursday, March 7.**

Read sections 1.1 to 1.5

(1) (1.1.8) Show that $\gamma(t) = (\cos^2(t) - \frac{1}{2}, \sin(t)\cos(t), \sin(t))$ is a parametrization of the curve of intersection of the circular cylinder of radius $\frac{1}{2}$ and the $z$-axis with the sphere of radius 1 and center $(-\frac{1}{2}, 0, 0)$. This is called *Viviani's Curve.*

(2) (1.2.3) A plane curve is given (in polar coordinates) by

$$\gamma(t) = (r(t)\cos(t), r(t)\sin(t)),$$

where $r(t)$ is a smooth function of $t$. Under what conditions is $\gamma$ regular? Find all functions $r(t)$ for which $\gamma$ is unit-speed. Show that, if $\gamma$ is unit-speed, the image of $r(t)$ is a circle; what is its radius?

(3) Compute the arc-length formula in polar coordinates (defined above).

(4) (1.3.2) The *cissoid of Diocles* is the curve whose equation in temrs of polar coordinates $(r, \theta)$ is

$$r = \sin\theta \tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

Note that this is a level curve description, versus the previous problems had a parametric curve description. Write down a parametrization of the cissoid using $t$ as a parameter and show that

$$\gamma(t) = \left(t^2, \frac{t^3}{\sqrt{1-t^2}}\right), \quad -1 < t < 1$$

is a reparametrization of it.

(5) (1.4.2) Give an example to show that a reparametrization of a closed curve need not be closed.

(6) (1.4.5) Suppose that a non-constant function $\gamma : \mathbb{R} \to \mathbb{R}$ is $T$-periodic for some $T \neq 0$. Show that there is a smallest positive $T_0$ such that $\gamma$ is $T_0$-periodic.