MATH 147A: HOMEWORK 1

Due Thursday, April 14.
Read Chapter 2

(1) (2.1.1) Compute the curvature of $\gamma(t) = (t, \cosh t)$.

(2) (2.2.1) Show that if γ is a unit-speed plane curve,
$$\dot{\mathbf{n}}_s = -\kappa_s \mathbf{t}.$$

(3) (2.2.5) Let γ be a regular plane curve and let λ be a constant. The parallel curve γ^λ of γ is defined by
$$\gamma^\lambda(t) = \gamma(t) + \lambda \mathbf{n}_s(t).$$
Show that if $\lambda\kappa_s(t) \neq 1$ for all values of t, then γ^λ is a regular curve and that its signed curvature is $\dfrac{\kappa_s}{|1-\lambda\kappa_s|}$.

(4) (2.3.1) Compute $\kappa, \tau, \mathbf{t}, \mathbf{n}, \mathbf{b}$ for
$$\gamma(t) = (\frac{1}{3}(1+t)^{\frac{3}{2}}, \frac{1}{3}(1-t)^{\frac{3}{2}}, \frac{t}{\sqrt{2}}).$$
and verify that the Frenet-Serret equations are satisfied.

(5) (2.3.6) Let (a_{ij}) be a skew symmetric 3×3 matrix. Let v_i, $i = 1, 2, 3$, be smooth vector functions of a parameter s satisfying the differential equation
$$\dot{v}_i = \sum_{j=1}^{3} a_{ij} v_j$$
for all i, and suppose that for some s_0, the vectors $\{v_1(s_0), v_2(s_0), v_3(s_0)\}$ are orthonormal. Show that the vectors $\{v_1, v_2, v_3\}$ are orthonormal for all s.

(6) Give an example of two space curves with the same curvature but are not isometric to each other (there is no isometry between them).