MATH 147A: HOMEWORK 4

Due Thursday, May 12.
Read Chapter 4

(1) (4.2.1) Show that, if $f(x, y)$ is a smooth function, its graph
$$\{(x, y, z) \in \mathbb{R}^3 \mid z = f(x, y)\}$$
is a smooth surface with atlas consisting of the single regular surface patch
$$\sigma(u, v) = (u, v, f(u, v)).$$

(2) (4.2.3) Which of the following are regular surface patches:
   (a) $\sigma(u, v) = (u, v, uv)$
   (b) $\sigma(u, v) = (u, v^2, v^3)$
   (c) $\sigma(u, v) = (u + u^2, v, v^2)$

(3) (4.2.7) Let γ be a unit-speed curve in $\mathbb{R}^3$ with nowhere vanishing curvature. The tube of radius $a > 0$ around γ is the surface parametrized by
$$\sigma(s, \theta) = \gamma(s) + a(\mathbf{n}(s)\cos(\theta) + \mathbf{b}(s)\sin(\theta)),$$
where $\mathbf{n}$ is the principal normal of γ and $\mathbf{b}$ is its binormal. Give a geometrical description of this surface. Prove that σ is regular if the curvature κ of γ is less than a^{-1} everywhere.

(4) (4.4.1) Find the equation of the tangent plane of each of the following surface patches at the indicated points:
   (a) $\sigma(u, v) = (u, v, u^2 - v^2), \quad (1, 1, 0)$
   (b) $\sigma(r, \theta) = (r\cosh\theta, r\sinh\theta, r^2), \quad (1, 0, 1)$

(5) (4.4.3) Let S be a surface, let $p \in S$ and let $F : \mathbb{R}^3 \to \mathbb{R}$ be a smooth function. Let $\nabla_S F$ be the perpendicular projection of the gradient ∇F of F onto T_pS. Show that, if γ is any curve on S passing through p when $t = t_0$, say
$$(\nabla_S F) \cdot \dot{\gamma}(t_0) = \left.\frac{d}{dt}\right|_{t=t_0} F(\gamma(t)).$$
Deduce that $\nabla_S F = 0$ if the restriction of F to S has a local maximum or a local minimum at p.