

MATH 147A: HOMEWORK 5

Due Thursday, May 19.

Read Chapter 5 and 6

- (1) (6.1.1) Calculate the first fundamental forms of
 - (a) $\sigma(u, v) = (\sinh(u) \sinh(v), \sinh(u) \cosh(v), \sinh(u))$.
 - (b) $\sigma(u, v) = (u - v, u + v, u^2 + v^2)$.
 - (c) $\sigma(u, v) = (\cosh(u), \sinh(u), v)$.
 - (d) $\sigma(u, v) = (u, v, u^2 + v^2)$.
- (2) (6.3.1) Show that every local isometry is conformal. Give an example of a conformal map that is not a local isometry.
- (3) (6.3.4) Let $\Phi : U \rightarrow V$ be a diffeomorphism between open subsets of \mathbb{R}^2 . Write

$$\Phi(u, v) = (f(u, v), g(u, v))$$

where f and g are smooth functions on the uv -plane. Show that Φ is conformal if and only if either $(f_u = g_v$ and $f_v = -g_u)$ or $(f_u = -g_v$ and $f_v = g_u)$.