Homework 4 Question 27 Solution

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It takes George 1 hour longer to mow the lawn than it takes Henry. Working together, using two mowers, they can mow the lawn in 1 hour and 12 minutes. How long would it take Henry to mow the lawn by himself? [Hint: Let x be the time taken by George. How much of the lawn does George mow in 1 hour? How much does Henry do in 1 hour? How much do they mow together in 1 hour?]

SOLUTION: Let's follow the hints step by step.

1. Let x be the time taken by George.

   In other words, In x hours, George will mow 1 lawn.

2. How much of the lawn does George mow in 1 hour?

   Before we do this, let's think of an example. Let's say we take 5 hours long to complete 1 test. Now let's ask ourselves, how much of the test can we complete in 1 hour? If one test contains 10 questions, and it takes us 5 hours to complete it, then in 1 hour, we should be able to do

   $$\frac{1 \text{ test}}{5 \text{ hours}} = \frac{10 \text{ questions}}{5 \text{ hours}} = 2 \text{ questions per hour}$$

   Going back to the original problem, George took x hours to mow 1 lawn. So in 1 hour, he should be able to do

   $$\frac{1 \text{ lawn}}{x \text{ hours}} = \frac{1}{x} \text{ lawn per hour}$$

3. How much does Henry do in 1 hour?

   The question states that George takes *1 hour longer* than Henry to mow the lawn. Then if George takes x hours, Henry should mow the lawn in $x - 1$ hours. So in 1 hour, he should be able to do

   $$\frac{1 \text{ lawn}}{(x-1) \text{ hours}} = \frac{1}{x-1} \text{ lawn per hour}$$

4. How much do they mow together in 1 hour?

   Since George mows $\frac{1}{x}$ and Henry mows $\frac{1}{x-1}$ in 1 hour, together, they will mow the sum of their rates:

   $$\text{combined rate} = \frac{1}{x} + \frac{1}{x-1} \text{ lawn per hour}$$

5. We have now exhausted all of the hints.

We now know the combined rate at which Henry and George mow the lawn. From the problem, we also know that in 1 hour 12 minutes, or $\frac{72}{60}$ hours, they mow 1 lawn. With this equation, we this becomes just another $D = RT$ problem!

D=1 lawn
R = $\frac{1}{x} + \frac{1}{x-1}$ lawn per hour
T = $\frac{72}{60}$ hour

Inputting these values into $D = RT$, we get

$$1 \text{ lawn} = (\frac{1}{x} + \frac{1}{x-1} \text{ lawn per hour}) \times (\frac{72}{60} hour)$$

and we can solve for x:

$$(x \cdot (x-1)) \cdot 1 = (\frac{1}{x} + \frac{1}{x-1}) \cdot (\frac{72}{60}) \cdot (x \cdot (x-1))$$

$$(x \cdot (x-1)) = (\frac{1}{x} \cdot (x \cdot (x-1)) + \frac{1}{x-1} \cdot (x \cdot (x-1))) \cdot (\frac{72}{60})$$

$$x \cdot (x-1) = ((x-1) + x) \cdot (\frac{72}{60})$$

$$x \cdot (x-1) = (2x-1) \cdot (\frac{72}{60})$$

$$(\frac{60}{72}) \cdot x \cdot (x-1) = (2x-1)$$

$$(\frac{60}{72}) \cdot (x^2 - x) = (2x-1)$$

$$(\frac{60}{72}) \cdot (x^2) - (\frac{60}{72})(x) - 2x + 1 = 0$$

$$(\frac{60}{72}) \cdot x^2 - (\frac{60}{72} - 2) \cdot x + 1 = 0$$

Because we have x to the power of 2, we *must use the quadratic formula.*

$$x = \frac{(\frac{60}{72} - 2) \pm \sqrt{(\frac{60}{72} - 2)^2 - 4 \cdot \frac{60}{72}}}{2 \cdot \frac{60}{72}}$$

and we get $x = .4$ hours and $x = 3$ hours. Since x is the time George takes, we need to subtract 1 hour to get Henry's time. Time is positive, so we use $x = 3$ hours. Then Henry takes 2 hours to mow the lawn.

ANSWER: 2 hours